Modelling Rule- and Experience-Based Expectations Using Neuro-Fuzzy Systems

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Abstract

Expectations modelling in macroeconomic theory is often done under restrictive assumptions regarding people's ability to learn and the level of their knowledge. This approach assumes that people know something about economic dependencies but that they are not informed of the exact formulas. Fuzzy rules represent the people's vague knowledge about the economic system. Neuro-methods are applied to train this knowledge. Both techniques are hybridized as a neuro-fuzzy system called the "Neuro-Fuzzy Expectation Generator (NFEG)". This module is connected to a business cycle simulation model using MAKROMAT-nfx. The software allows us to analyze how the NFEG interacts with the economic system when the later is exposed to exogenous shocks.

JEL-Classification: C6, C8, E3

Contents

1. Introduction **1**

2. The Business Cycle Simulation Framework **3**

2.1 Building Blocks and their Interconnections 3

2.2 Simulations Using Conventional Expectation Hypotheses 5

3. Using and Training Expectation Rules **8**

3.1 Step 1: Design of the Fuzzy Expectation Generator...8

3.2 Step 2: Acquiring Experiences by means of Neural Learning Methods.....................11

3.3 Simulation Studies...16

4. Applications and Prospects **22**

1. Introduction

Expectations modelling in macroeconomic theory is often done under restrictive assumptions regarding people's knowledge level and learning ability. Either it is assumed that people do not learn at all, which justifies the use of simple autoregressive forecasting methods, or the model makers believe that the relevant agents know everything about the (long-term) behavior of the economic system (rational expectations). The first option implies regular oscillations each time the economy's equilibrium is disturbed by exogenous shocks. These oscillations repeat themselves mechanically because the economic units cannot fall back on any experience and knowledge about the economic process. The use of rational expectations on the other hand renders the economic process almost completely resistant to exogenous shocks, so their effects - if there are any at all - last for a very short time only.

Neither of these modelling options seem realistically to describe what people really do in anticipating future developments when making current decisions. The lack of an adequate expectations model is especially problematic in business cycle theory where expectations play a dominant role in the cyclical behavior of main macroeconomic indicators.

The working hypothesis of this paper is that economic units are able to learn from their experiences to a certain extent. The knowledge won in such a way is not sufficient however to understand the complex reality completely (bounded rationality). They are rather faced with a complex system over whose operation assumptions in form of vague rules are set up. A typical expectation rule could be: „With normal inflation so far, due to the low unemployment rate in the future a somewhat stronger upward trend of prices is expected.“ These rules must be flexible in the sense that the scopes of the indistinct terms like ‚normal inflation‘ or ‚low unemployment‘ depend upon the values of the respective variables actually observed in the past. E. g. nowadays something else is understood by ‚low unemployment‘ than twenty years ago.

Vague rules are characteristic of fuzzy systems. Adaptability distinguishes artificial neural networks. Therefore, it seems worth trying to combine both techniques. In order to get an operable expectation model, fuzzy rules are formulated which are transformed into an equivalent neuronal network for training purposes. Afterwards the neural net is retransformed into the original fuzzy structure. This hybrid neuro-fuzzy system is called the „Neuro-Fuzzy Expectation Generator (NFEG)“. From the model constructor‘s point of view it is indispensable to keep the whole system economically understandable. This forbids the application of purely neural systems which might even obtain a higher learning performance. Due to their black box character however they are model-theoretically completely unsuitable. Furthermore the application of neural learning procedures must not degenerate the fuzzy system. For this reason a modified error backpropagation algorithm is presented in this paper, which allows for readjusting fuzzy parameters when they tend to quit the economically reasonable ranges during the learning process (Controlled Error Backpropagation (CEBP)).

To contrast this new approach with conventional concepts it is called „rule- and experience-based expectations“ (REB-expectations). The working of the suggested technique cannot be illustrated by pure formulas only. It needs a simulation environment which has been realized by developing the macroeconomic simulation software MAKROMAT-nfx[1]. This program integrates the NFE-Generator into a macroeconomic business cycle model. Within this environment it is possible to expose the model economy to exogenous shocks and to observe how different expectation concepts (autoregressive, rational or rule- and experience-based) influence the adjustment processes.

In the next paragraph the business cycle model used for simulation studies is sketched. Since the underlying set of equations is documented completely by the software itself, the description concentrates on the main building blocks. This stylized outline permits a first rough insight into the dynamic characteristics of the model. Its explicit dynamic behavior is shown by two simulation studies which serve as a reference for further work with the NFE-Generator.

The third paragraph describes the theoretical conception of the NFE-Generator as well as its performance when it is connected to the business cycle model. The neuro-fuzzy system is developed in two steps. First a short introduction into fuzzy logic fundamentals is delivered. This allows us to see the parallels between technical fuzzy controllers and economic fuzzy expectation generators. Second the learning algorithm and the conversion of the fuzzy system into an equivalent neural network is discussed following the basic ideas of Lin and Lee[2]. After presenting the necessary refinements of the original error backpropagation algorithm the simulation studies of paragraph two are picked up in order to show the effect of two different types of rule- and experience-based expectations.

The papers ends up with a glance at possible applications of the NFE-Generator and some suggestions for further research work.

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[1] The software has been designed for Microsoft Windows NT 4.0 and Windows 98. The installation routine can be downloaded free of charge visiting the MAKROMAT Webcenter’s special site dedicated to this paper (http://makromat.uni-muenster.de?cef99). Here only a small part of the software‘s range of options can be presented. The reader is encouraged to explore the software by doing experimental work with it on his/her own.

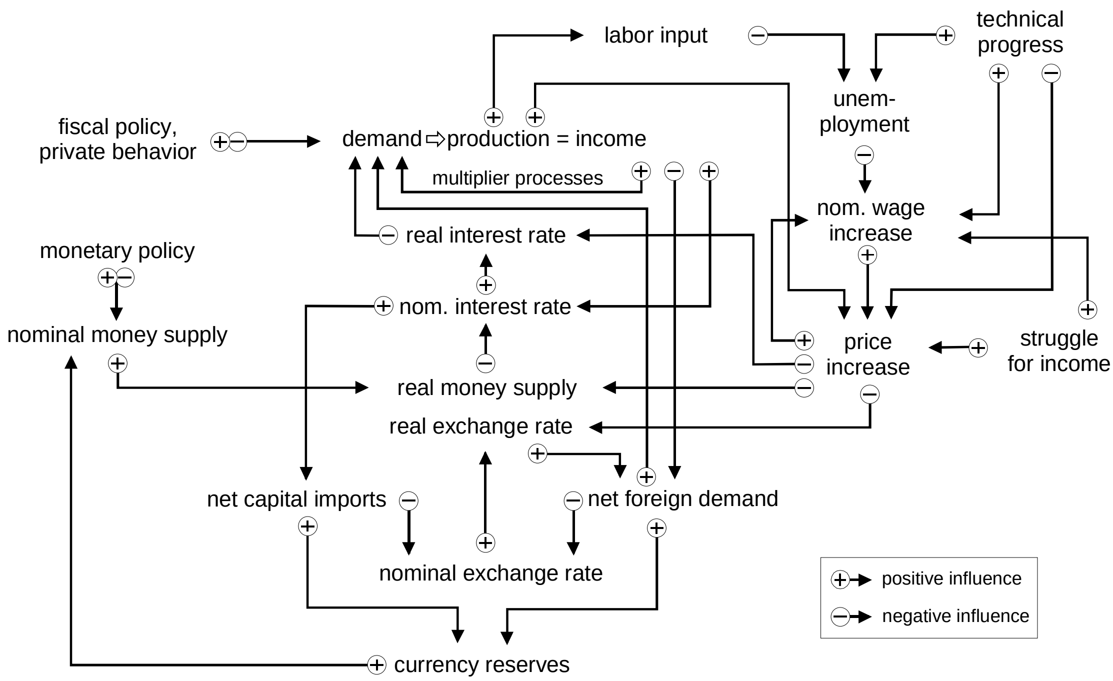
[2] See Lin/Lee (1991) and, in more detail, Lin (1994).

2. The Business Cycle Simulation Framework

2.1 Building Blocks and their Interconnections

The model used for simulation purposes explicitly considers the goods market, the foreign exchange market (flexible exchange rate system), the securities market (i.e. money and capital market) and the labor market. Hereby the inflation-income-interest connection is given special emphasis.[3] The underlying time horizon is short- and medium-term oriented, so growth effects remain out of focus. Fig. 1 demonstrates the model's mode of operation which results from the formulated behavioral equations for the various sectors.

Fig. 1 — *Mode of operation (direct effects only)*



Roughly speaking the connections within the model can be sketched as follows: Firms produce according to their expected sales position, so there is a close link between production (X) and demand for goods (N). Price setting is based on unit labor costs (markup pricing) with the markup factor being positively influenced by the rate of capacity utilization. As long as the capacity utilization is constant, changes in nominal wages (gwN)[4] are fully passed along to prices. Labor productivity growth (ga) reduces inflation whereas the struggle for income (represented by the autonomous markup factor j_V) stokes it up. All direct factors put together, the inflation rate (gP) follows equation (1):

(1) $$gP = gwN - ga + j_V$$

3 The fundamentals of the model can be found in DORNBUSCH/FISCHER (1994), Ch. 16, and GORDON (1993), Ch. 9. The interest rate dynamics result from a market clearing approach which is related explicitly to the entire market for securities (money and capital market; for the underlying idea see KOOTHS (1994).

4 The anteposition of „g“ identifies growth rates whereas a postponed „N“ is used to mark nominal values.

While ga and j_V are exogenous quantities, the growth rate of nominal wages is further explained within the model. Its determing factors are (1^{st}) compensation for expected inflation (the expected inflation rate is passed along to nominal wages by w_{gP}·100 %), (2^{nd}) the workers market power (covered by confronting the actual unemployment rate u to its natural level u^{nat} and weighting the resulting difference with the parameter w_u), (3^{rd}) exogenous wage increases (w_V) reflecting the workers' attempts to shift their share in national income upwards and finally (4^{th}) the workers participation in technical progress expressed by means of the parameter w_{ga}. These four factors result in the following wage development function:

$$(2) \qquad gwN = w_{gP} \cdot gP^{exp} + w_u \cdot (u^{nat} - u) + w_V + w_{ga} \cdot ga$$

Substituting gwN in (1) by (2) and converting u into X via a linear-limitational production function ($X = a \cdot A$, with A and A^{pot} denoting the employed and the potential labor force respectively) we get the modified expectation augmented phillips curve for explaining the supply-side roots of inflation:

$$(3) \qquad gP = w_{gP} \cdot gP^{exp} + \frac{w_u}{a\,A^{pot}}(X - X^{nat}) + (j_V + w_V) + (w_{ga} - 1) \cdot ga$$

Isolating X in (3) delivers the aggregate supply function:

$$(4) \qquad X = X^{nat} + \varphi \cdot [gP - w_{gP} \cdot gP^{exp} - (j_V + w_V) + (1 - w_{ga}) \cdot ga] \quad \text{with:} \quad \varphi = \frac{a \cdot A^{pot}}{w_u}$$

The demand side of the economy is influenced by the following factors: (1^{st}) exogenous fluctuations in aggregate demand (ΔN^{aut}) reflecting e. g. a change in fiscal policy, (2^{nd}) the interest-sensitive demand response to monetary policy resulting from the realized central bank's nominal monetary growth target (gMN) and its implicit real money supply effect (gMN – gP) on the level of nominal interest rates (i), (3^{rd}) a change in the expected rate of inflation (ΔgP^{exp}) modifying the expected level of real interest rates and therefore influencing interest-sensitive demand. Starting out from the previous period and measuring the resulting effects of the factors above by means of their respective income multipliers taken from the underlying IS-LM-Z-model the level of aggregate demand in the current period can be expressed as follows:

$$(5) \qquad N = N_{-1} + \mu_{N^{aut}} \cdot \Delta N^{aut} + \mu_{gM} \cdot (gMN - gP) + \mu_{gP^{exp}} \cdot \Delta gP^{exp}$$

The equilibrium condition (6) joins the demand- and the supply-side of the economy:

$$(6) \qquad X \overset{!}{=} N$$

If the unions get accepted a complete compensation for inflation ($w_{gP} = 1$), then the long-term level of income is fixed at the natural production level X^{nat} whatever the inflation rate may be. In the long run the later equals the nominal monetary growth rate. For getting a first insight into the dynamic behavior of this model, let us assume the existence of such an equilibrium in period zero (t = 0). One period later the central bank decides to change the former nominal monetary growth rate durably ($\Delta gMN_1 \neq 0$). Assuming further that the economic units form their inflation expectations in a simple adaptive way ($gP^{erw} = gP_{-1}$) and regarding (4), (5) and (6) simultaneously, the resulting income course can be described by the following difference equation:[5]

$$(7) \qquad Y_t = X_t = X^{nat} + \left(\sqrt{\frac{\varphi}{\mu_{gM} + \varphi}}\right)^t \cdot \left[\sqrt{\varphi \cdot \mu_{gM}} \cdot \Delta gMN_1 \cdot \sin\left(t \cdot \sin^{-1}\left(\sqrt{\frac{\mu_{gM}}{\mu_{gM} + \varphi}}\right)\right)\right]$$

One gets an oscillating process around the natural production level X^{nat} with a constantly diminishing amplitude. As long as inflation expectations follow the simple adaptive procedure each monetary policy change would generate exactly the same swinging adjustment process. In the case of rational (i.e. model-consistent) expectations ($gP^{erw} = gMN$) a monetary policy change would not have any cyclical effect at all if the economic units are immediately informed about shifts in the growth rate of nominal money supply.

Because of some strongly simplifying assumptions the regular adjustment process to (monetary) disturbances of the inflation-income-interest equilibrium that can be seen from the difference equation (7) should only be considered as a first stylized approximation. Especially the use of IS-LM-Z-multipliers implies that income adjustment processes triggered off by non-income induced changes in aggregate demand come to an end before wages are renegotiated (the time index t denotes the period during which nominal wages and prices are constant). In fact these processes normally take more time which constitutes the need for a temporally more differentiated analysis. Linking the multiplier dynamics and the dynamic phillips curve mechanism renders the resulting algebra unsolvable. For this reason the macroeconomic simulation software MAKROMAT-nfx is used in the next paragraph to show the business cycle dynamics when both processes are treated explicitly.

2.2 Simulations Using Conventional Expectation Hypotheses

Let us consider an interval of 600+1 periods for the following simulation purpose. Now the length of each basic period is determined by the time while the goods demanders‘ income expectations and the goods suppliers‘ demand expectations are constant, so each period corresponds to one round in the income multiplier process. It is assumed that wage bargainings take place at the beginning of every fourth period (length of wage period = four basic periods). For clarity reasons let the time horizon of the central bank’s monetary policy be equal to the wage period (i.e. constant nominal money supply during four basic periods). We are again starting from an equilibrium situation in period zero. Beginning in t = 1 this equilibrium is disturbed eight times during the whole interval because of monetary policy changes (variations of the nominal monetary growth target). Each monetary policy constellation is maintained during 75 basic periods in order to give the adjustment processes enough time to work. The case of

[5] The fact that the income path can be calculated by means of a difference equation should not mislead us to believe that people can use this formula for their income and inflation expecations because equation (7) does only describe a stylized development and it contains quantities that are unknown to the economic units.

adaptive inflation expectations can be seen in Fig. 2 (corresponding model file: „demo-nfx1 (adaptive).mm5“)[6].

Fig. 2 — *Income fluctuations in response to monetary policy switches (adaptive expectations)*

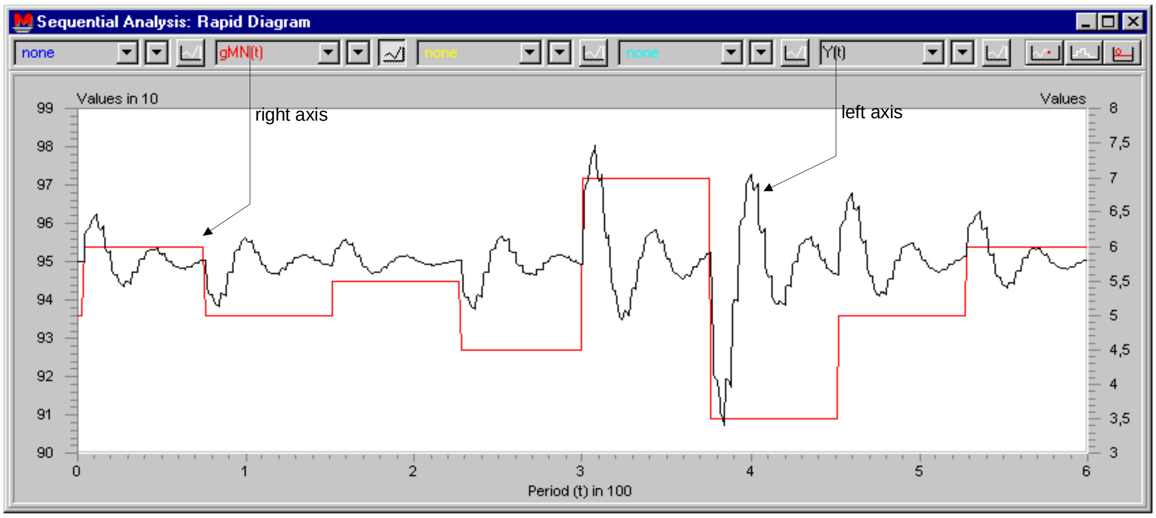


Fig. 3 — *Income-inflation adjustment paths for all periods (left) and those with wage bargaining only (right)*

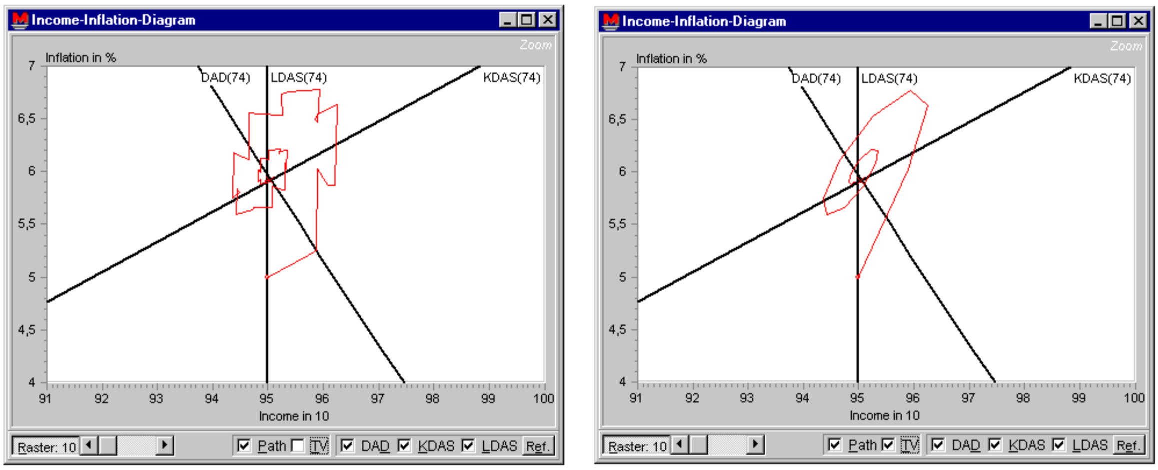
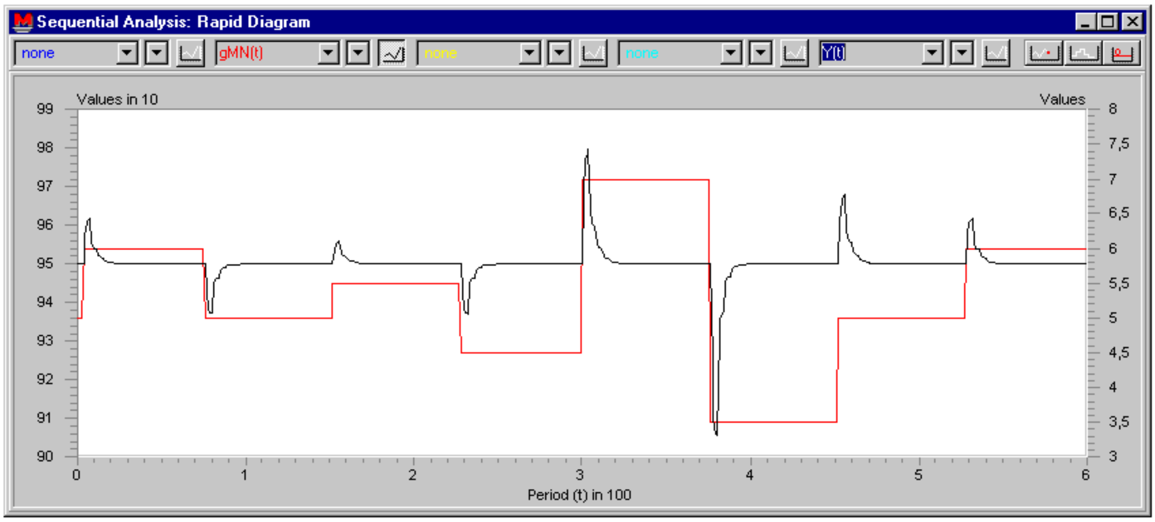


Fig. 2 reveals the typical cyclical adjustment processes in response to monetary disturbances. While the amplitudes depend upon the impulse effected by the monetary policy switch (the larger the change in the nominal money supply growth rate the heavier the respective shock) the process pattern always follows the same simple regularity. To demonstrate this stereotyped adjustment process in another way, Fig. 3 shows the helical pattern generated for the first 75 periods in an income-inflation diagram. If only the periods with wage bargaining activities are displayed (right diagram in Fig. 3) we get a pattern which is very similar to that which would be created using the sterile difference equation (7).

[6] The relevant files for all simulations presented here have been compiled for free downloading on this paper’s special website (see above).

The case of rational expectations ($gP^{erw} = gMN_{-1}$) is shown in Fig. 4 (file: „demo-nfx1 (rational).mm5“). There are two reasons why monetary policy switches lead to (temporary) real income effects despite of the rational expectations hypothesis: (1^{st}) the existing recognition lag of one period gives the central bank rope for short-term deception tactics and (2^{nd}) the expectations are rationally designed with regard to the long run model-consistent outcome only; since the money demand in the economy is sensitive to changes in the nominal interest level, long run rational expectations underestimate (overestimate) the short term inflation developments effected by expansionary (restrictive) monetary policy measures because of the contraction (expansion) of real money supply involved herewith. Fig. 4 shows that rational expectations do not protect the economy from regular short-term real adjustment effects of nominal monetary policy measures. Attention should be paid to the fact that the economic units maintain their monocausally founded inflation forecast despite the poor predictive power of this hypothesis during the first periods of each adjustment process.[7]

Fig. 4 — *Income fluctuations in response to monetary policy switches (rational expectations)*



As stated at the beginning and demonstrated by the preceding exemplary simulations both traditional expectations hypotheses are not really convincing. In the following paragraph a procedure is presented which aims at finding a way between the extremes of rational and autoregressive expectations without discarding their respective plausible characteristics: foundation of expectations upon a theoretical background and observation of the past in order to identify and learn from forecast errors. Therefore this new approach is called „Rule- and Experience-Based Expectations“. Its technological background consists in a hybrid neuro-fuzzy system whose structure and operational mode is explained next.[8]

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[7] E. g. after the expansionary shock in period t = 300 the inflation rate amounts to 10,10 %, 9,39 % , 8,98 % in period 304, 305 and 306 respectively. These values diverge considerably from the new equilibrium level of 7 %.

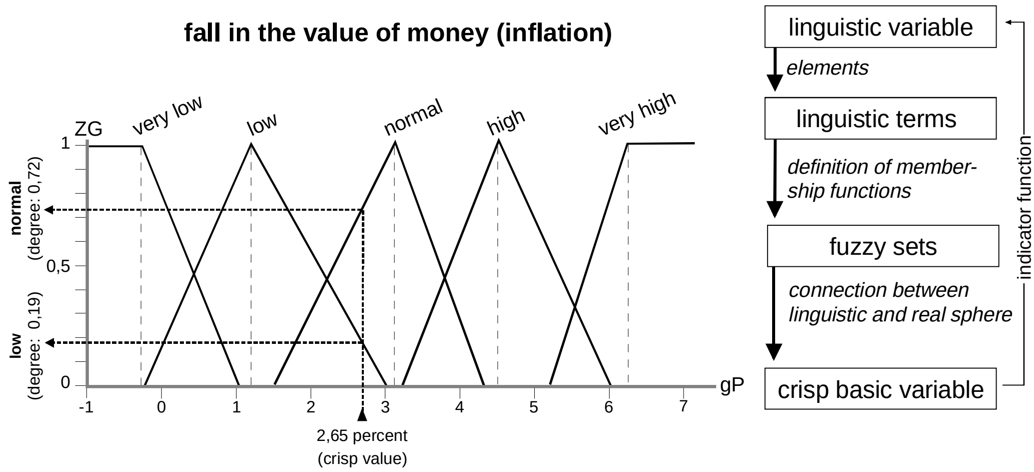
[8] Only a short outline of the applied neuro-fuzzy technology can be given here. For details and alternative options see KOOTHS (1998), Ch. 2.

3. Using and Training Expectation Rules

3.1 Step 1: Design of the Fuzzy Expectation Generator

The first step towards the Neuro-Fuzzy Expectation Generator consists in formulating the theoretical background people use for forecasting in an operational way. This is realized by a system of fuzzy rules (Fuzzy Expectation Generator).[9] The FE-generator calculates forecasts based on observable time series (past data). Inputs and outputs are crisp values whereas the embodied knowledge is represented on a linguistic level. This allows explicitly for a vague and therefore rather realistic estimation of the economic situation and the expectation formation process resting on it.

Fig. 5 — *Fuzzifying the inflation rate*



Since the inflation rate plays a predominant role for both the supply- and demand-side in the presented macroeconomic simulation framework, it is used here for an exemplary demonstration of REB-expectations. In its fuzzified form the expected quantity possesses five vague partitions (‚very low‘, ‚low‘, ‚normal‘, ‚high‘ and ‚very high‘) called ‚linguistic terms‘ which in the present example represent the elements of the linguistic variable ‚fall in the value of money (inflation)‘. The link between these linguistic terms and the crisp inflation data is realized by defining fuzzy sets via membership functions.[10] The membership function of each fuzzy set indicates to what degree any crisp inflation rate is associated with the respective linguistic term (degree of membership: MD). The example in Fig. 5 shows a crisp inflation rate of 2.65 % which is regarded as ‚low‘ to a degree of 0.19 and as ‚normal‘ to a degree of 0.72

[9] For an introduction into fuzzy logic see ZIMMERMANN (1993). The classical reference to the roots of fuzzy set theory is ZADEH (1965).

[10] The NFE-generator works with triangular or bell-shaped fuzzy sets. Both types are characterized by the same parameter vector: the center indicates the representative value of the respective term whereas the width of a fuzzy set determines its extent (asymmetrical fuzzy terms have a left and a right width). In order to cover an open sharp interval completely triangular fuzzy sets must be designed with border saturation (i.e. infinitely great external widths). This kind of modelling is optional for bell-shaped fuzzy sets.

(degrees of membership do not need to total up to one because it is the relative membership that actually matters).

Those variables which people assume to influence the expected quantity are referred to as determinants. The question how many and what kind of determinants are used to explain the expected quantity must be answered exogenously. This example works with two determinants: at the beginning of each basic period people forecast the inflation rate by founding their expectations on the current unemployment rate and the observable money growth rate. These determinants imply that people believe in a hybrid inflation explanation which regards supply-side reasons (labor market situation) as well as demand-side causes (monetary policy) simultaneously. To keep the example as clear as possible only three linguistic terms are formulated for fuzzifying each determinant (‚low‘, ‚normal‘, ‚high‘). The connection between the linguistic terms of the determinants with those of the expected quantity is realized with fuzzy rules which follow a simple if-then-scheme. Fig. 6 elucidates the underlying principle of fuzzy inference with fuzzy rules. It concentrates on a singular 1-dimensional fuzzy rule expressing the phillips-curve idea that high unemployment usually comes along with low inflation.

Fig. 6 — *Fuzzy rules and fuzzy inference: basic principle*

a) stylized form of a fuzzy rule

fuzzy rule

IF condition(s) **THEN** reaction to condition(s)

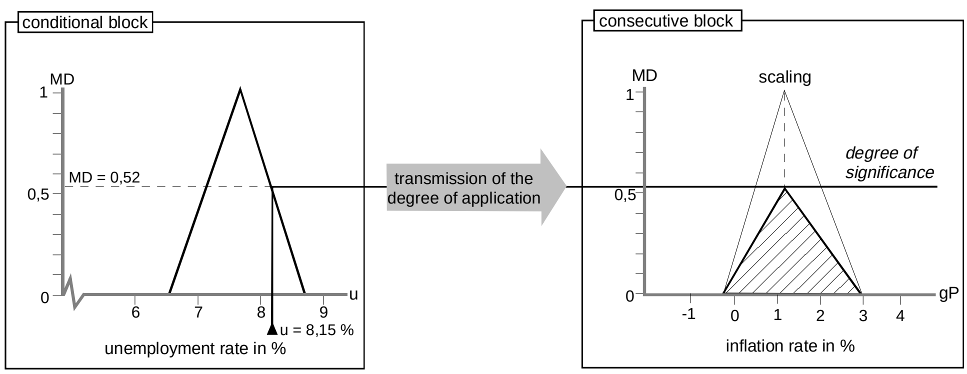
conditional block *consecutive block*

b) inference principle

conditional block → *transmission of the degree of application* → consecutive block

c) example (rule estimation by means of the scaling procedure)

IF (unemployment = "high") **THEN** (inflation = "low")

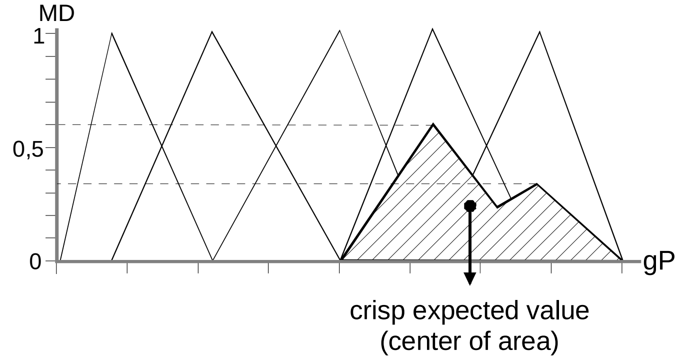


Unlike 0-1-implications of classical logic, fuzzy rules can also apply gradually. As illustrated by the example in Fig. 6 the degree of membership of the conditional block („the unemployment rate of 8.15% is considered high to a degree of 0.52“) determines the fuzzy rule’s degree of application which is transmitted as degree of significance to the consecutive block („inflation is expected to be low to a degree of 0.52“). If the scaling method is used to evaluate fuzzy rules the consecutive fuzzy set is compressed proportionally to its degree of significance.

The example shown in Fig. 6 represents the most simple case of a single mono-dimensional expectation rule. In order to realize the forecasting behavior described above (expected inflation depending on the labor market situation and monetary policy) fuzzy rules with multi-dimensional conditional blocks must be created (example: „IF the unemployment rate is high AND money growth is low THEN inflation is expected to be very low.“). In this case the degree of application depends on the simultaneous presence of ‚high unemployment‘ and ‚low money growth‘ which are connected with a fuzzy AND operator. The evaluation of such a multi-dimensional fuzzy rule is called aggregation. As a common aggregation method multiplication of the determinants‘ degrees of membership is used.

A single fuzzy rule is not enough to describe the variety of possible observations and judgements concerning the economic situation. Therefore a bundle of fuzzy rules for different combinations of unemployment and monetary policy is necessary which is called fuzzy rule base. Such a rule base delivers a significant inflation forecast for any observed value of unemployment and money growth. The present example with a two-dimensional rule base with each determinant possessing three linguistic terms, $3 \times 3 = 9$ fuzzy rules are required in order to cover all possible combinations. It is possible that two or more fuzzy rules shoot at the same consecutive term. This leads to an accumulation method for calculating the degree of significance of the consecutive term which is hit repeatedly. For this purpose the limited-sum procedure can be used which cumulates the degrees of application of the respective rules and limits the result to one in order to prevent degenerated consecutive fuzzy sets.

Fig. 7 — *Fuzzy inference result set and the center-of-area defuzzification procedure*



At the end of the fuzzy inference process we get a vague statement in the form of a fuzzy inference result set (= join of sets of all scaled consecutive fuzzy sets). With reference to the example of Fig. 7, a possible result could be that future inflation is regarded as ‘high’ to a degree of 0.6 and as ‘very high’ to a degree of 0.34. Such a statement does reveal all the vagueness and ambiguity people face in the forecast process but it is not suitable for concrete economic activities which require a condensed crisp result. If, for example, the wage rise is pegged to the rate of expected inflation, a crisp inflation forecast is needed as a basis for wage negotiations.

There are several defuzzification methods available whose common target consists in generating a representative value of the fuzzy inference result set. In Fig. 7 the center-of-area procedure was chosen to get the crisp result of the fuzzy inference process for inflation forecasting.

Given the rule base, the fuzzy expectation formation process can be summed up as follows:

(1) fuzzification of the determinants and the expected quantity (allocating crisp values to linguistic terms using membership functions)

(2) fuzzy inference
- aggregation (calculating the degrees of application of each expectation rule)
- accumulation (determining the degrees of significance of all consecutive terms)
- inference in the narrower sense (generating the fuzzy inference result set)

(3) defuzzification (finding a crisp representative result)

By analogy with technical systems (fuzzy control)[11] Fig. 8 shows the interaction between the fuzzy expectation generator and the economic model in which it is embodied.

Fig. 8 — *Fuzzy expectation formation as an economic feedback control system*

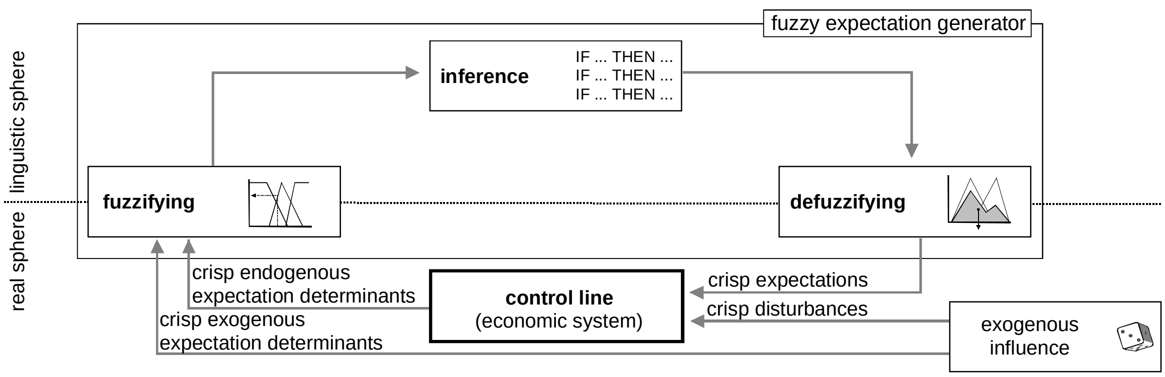


Fig. 8 hints at the weak point of pure fuzzy systems for expectation formation, because there is no way forecast errors can modify the forecasting process (missing error correction channel). Therefore it is necessary to extend the fuzzy expectation generator by learning methods which train the fuzzy system in response to forecast errors. The used technology is presented in the following paragraph.

3.2 Step 2: Acquiring Experiences by means of Neural Learning Methods

The second step consists in extending the fuzzy expectation generator to a hybrid neuro-fuzzy system (connectionist fuzzy control system) following the basic technology presented by Lin

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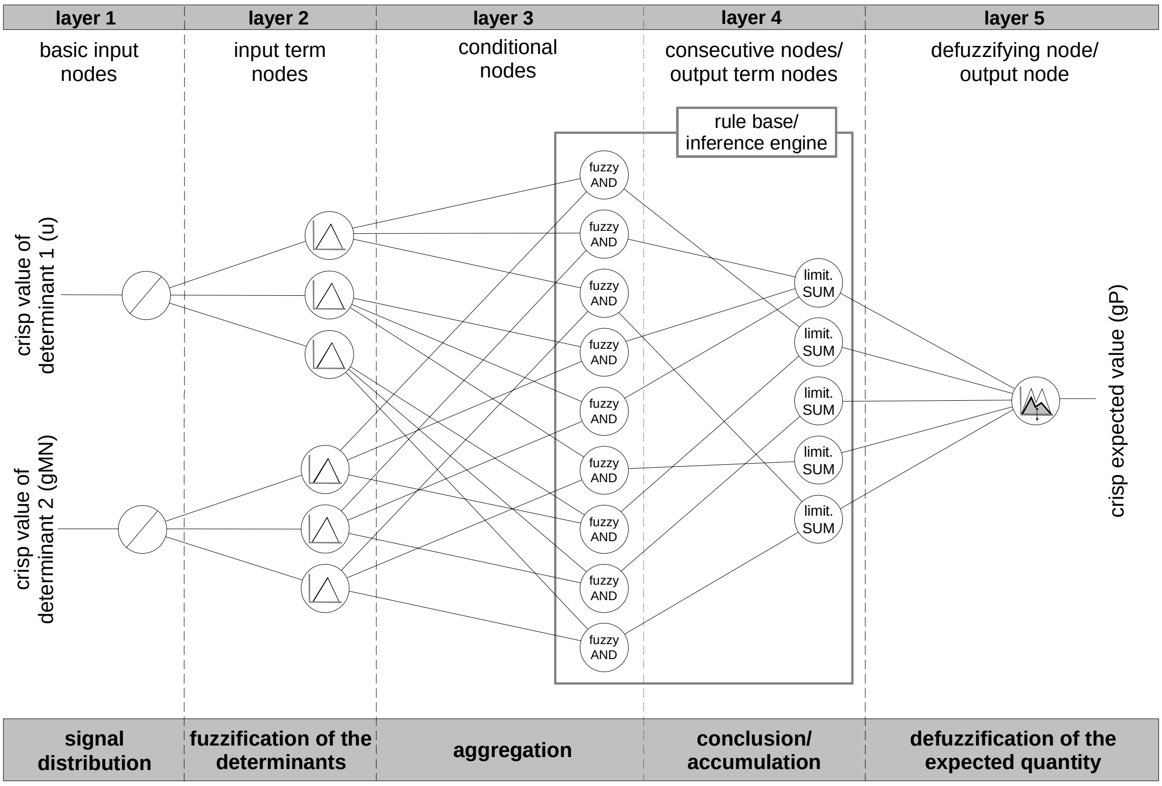
[11] The general functionality of a technical fuzzy controller is outlined in MAMDANI/ASSILIAN (1975).

and Lee.[12] This approach uses a layered feedforward neural net with a total of five layers (see Fig. 9). Each layer carries out a single function in the fuzzy inference process. Nodes at layer one (basic input nodes) are sensors to the outside world. Their task is to observe the crisp values of the determinants and transmitting them to the appropriate nodes in layer two (input term nodes) which fulfill the fuzzification function for each determinant. So in the present example we need two basic input nodes (one for the unemployment rate and the other for the money growth rate). Every basic input node is connected with three input term nodes each of them representing a different linguistic term of the respective determinant.

Fig. 9 — *Neuro-fuzzy system for expectation formation*



The parameters used for characterizing the membership functions (centers and widths of the fuzzy sets for the linguistic terms) can be interpreted as link weights between layer one and two. After calculating the degrees of membership for all linguistic determinant terms the layer two nodes propagate this output to the next layer whose nodes act as conditional blocks of the fuzzy rule base (conditional nodes). Each of them calculates the degree of application of the respective rule by performing the applied fuzzy AND operator. All possible cross term combinations between both determinants are represented in the aggregation layer, so the number of links of each conditional node to the anterior nodes equals the number of determinants. Since the aggregation procedure works with unweighted input data (i.e. degrees of member-

[12] See LIN/LEE (1991) und LIN (1994). A good overiew of hybridizing neuro and fuzzy technologies is delivered by NAUCK/KLAWONN/KRUSE (1994), pp. 231 ff. A very concise description of the basic concepts of neuronal networks can be found in BUCKLEY/FEURING (1999), Ch. 3. For details in neural technologies see Hecht-Nielsen (1991).

ship of the concerned terms) the link weights between layer two and three are constant and equal to one. Each node at layer four (consecutive nodes at the conclusion/accumulation layer) corresponds to one linguistic term of the expected quantity. Therefore we find five nodes at layer four. Since the expected quantity is the output of the whole net we could also denominate these neurons as output term nodes. Each of these nodes receives the degrees of application of those conditional nodes which shoot at the respective consecutive term represented by the considered node at layer four. Optionally the degrees of application can be weighted which is then realized by means of link weights between the layers three and four. Each conditional node is connected with one consecutive node only. The opposite case does not necessarily apply: it is possible that one consecutive node is hit by no, one or more than one link. In the last case the consecutive node executes the limit-sum procedure for calculating the degree of significance for the consecutive term that it represents. The third and fourth layer together constitute the connectionist inference engine which embodies the whole fuzzy rule base of the equivalent fuzzy system. The single node at the fifth layer (output node) serves as defuzzifier and delivers the crisp forecast value. The link weights at layer five represent the centers and widths of each term of the expected quantity.

From a neural point of view the efficiency of a net with a given structure and node functionality does only depend upon the values of the link weights which determine how the node output of layer s is propagated to the subsequent nodes in layer s+1. The knowledge of a neural net is therefore embodied in the values of the link weights. The fact that the complete functionality of the fuzzy inference process is represented equivalently by the neural net allows the application of neural learning methods. Here the (modified) error-backpropagation procedure is used for training the rule based expectation formation process. The starting point of the learning process is the forecast error Err. For the present inflation expectation problem, Err can be developed as:

$$(8) \qquad \mathrm{Err} = \frac{1}{2}\frac{1}{np}\sum_{p=1}^{np}(gP_p - gP_p^{erw})^2$$

The epoch size np indicates the number of learning patterns from former periods. In the easiest case the last np periods are used for training purposes (learning at the current border).

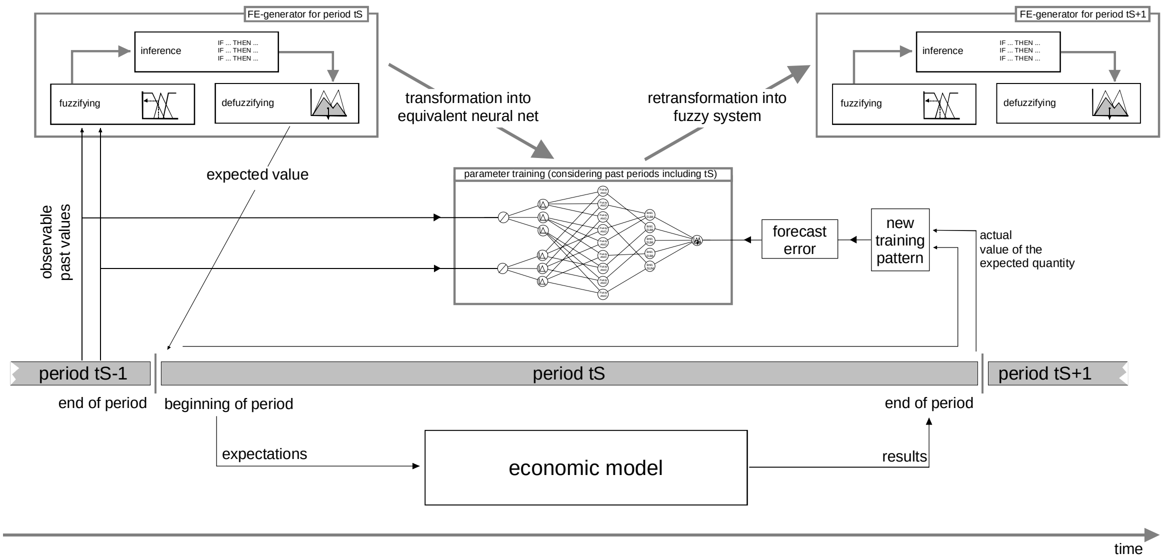
The goal of the learning procedure consists in minimizing the error function (8) on the weight space by finding the weight vector which minimizes Err. The idea behind the error-backpropagation procedure is that all nodes of the net (not only the output node) are responsible for the network error since they all influence the result by modifying the signal that is propagated through the network. During the training phase the signal's direction is reversed, so each learning round starts at the output node with the network error Err being fed into the network. This error signal is then backpropagated layer by layer until it reaches the basic input nodes. Hereby, the global network error is distributed over all relevant nodes. Because the link weights are the only adjustable parameters they are the central object of the learning process. The adjustment of each weight g is done proportionally to its marginal influence on the network error. This method implies a linear approximation of the error function in the environment of the current weight values.

(9) $$\Delta g = -\mu \frac{\partial Err}{\partial g}$$

Plotting the network error as a function of all link weights we get a mountain-like error surface in which, according to equation (9), the steepest way down is chosen with the learning rate μ determining the stride (gradient descent algorithm). We omit the details and formulas of the parameter adjustment procedures here,[13] but it should be noticed that they depend on the used term shapes, the fuzzy AND operator and the rule weighting option.

After each simulated period tS we get a new training pattern for the learning process, so the latter can be repeated with new learning material after each period as sketched in Fig. 10.

Fig. 10 — *Sequence of rule-based forecasting and experience-based expectation training*



In contrast with pure neural approaches the forecast output of this neuro-fuzzy system can be fully understood and interpreted. This is due to the fact that the nodes of two adjacent layers are not fully interconnected. The missing linkages suggest that this network type is less precise than a fully interconnected one. This is the price for the model maker's desire to get an interpretable system (trade-off between transparency and efficiency). The implementation of a fully interconnected reference network for efficiency comparisons is an enlargement for future releases of the simulation software.

The conflict between transparency and efficiency occurs during the learning process as well. It is sometimes appropriate, not to adapt certain parameters or to restrict their adjustment ranges, in order to keep the whole fuzzy system in a sound state. Otherwise it is possible that the standard backpropagation algorithm ruffles the fuzzy sets leaving us with a degenerated system that is no longer economically understandable. For this reason MAKROMAT-nfx was

[13] See KOOTHS (1998), section 2.5.3.4.

equipped with a „controlled error-backpropagation“ algorithm which differs from the standard version in the following (mostly optional) fields:

- Perception intervals (quasi-optional)
  minimum level of perception of crisp values
  ⇨ signal reinforcement in the case of extreme runaways for bell-shaped fuzzy sets without border saturation
  ⇨ signal attenuation for fuzzy sets with border saturation
- Melting of border saturation (optional)
  parameter adjustment for $\partial Err/\partial g = 0$
  ⇨ enables fuzzy sets to follow the basic variable when the latter enters „virgin soil“
- Selective parameter training (optional)
  symmetrical fuzzy sets, suppression of width and/or rule weight training
  ⇨ enhances the generalization capacity by means of parameter reduction
- Suppression of negative weights
  positive fuzzy set widths and rule weights
  ⇨ indispensable for a workable system
- Center limitation to actual extreme values (optional)
  term centers are limited to actually observed values of the basic variable
  ⇨ enhances a more regular partitioning of the observed crisp interval with fuzzy sets
- Relative limitation (optional)
  relative maximum for parameter adjustment at each training step
  ⇨ increases learning inertia and counteracts the influence of runaways
- Conservation of the rule base (optional)
  swapping of term centers is prevented
  ⇨ allows economic units to maintain a firm theoretical background
- Maximum term overlapping (optional)
  upper limit for the degree of membership at which two terms intersect by means of restricted center and/or width adjustments
  ⇨ works against degenerated partitioning and raises the selectivity of adjacent fuzzy sets
- Minimum adjacent term center overlapping (optional)
  lower limit for the degree of membership at the term centers of the adjacent fuzzy sets
  ⇨ reduces „fallow land“ between fuzzy sets and enhances a more regular partitioning
- Minimum term overlapping for triangular fuzzy sets
  lower limit for the degree of membership at which two triangular fuzzy sets intersect
  ⇨ reduces „fallow land“ between fuzzy sets and enhances a more regular partitioning
- Absolute minimum level for term widths
  obligatory lower limit for all term widths: 0,0001
  ⇨ prevents punctiform fuzzy sets and maintains computability of the system

The controlled error-backpropagation algorithm was especially designed for those situations in which the economic units find themselves on „virgin soil“ because of a radical change within the economic system. „Virgin soil“ appears whenever the fuzzy sets do not cover the relevant crisp interval in an adequately differentiated way and/or whenever the relevant crisp values lie in the border regions of the fuzzified interval. The CEBP-algorithm copes with such situations by different methods (see the above list) which all aim at rearranging the fuzzy sets in order to get a differentiated partitioning. The underlying idea of those methods is the formulation of various criteria for a sound expectation rule system and to inhibit the learning process whenever one (or more) of these criteria is running the risk to be violated.

Each economic time series simulation that aims at modelling learning processes faces the initialization problem, i.e. the question how to treat the foreknowledge of the economic units. For this problem the simulation software offers the following three options:

(a) Individual explicit input of all fuzzy parameters including the rule base

(b) Fully automatic clustering of a reference interval; the rule base can be generated semi-automatically by indicating the ceteris paribus influence of each determinant

(c) Unsupervised competitive pre-learning phase for clustering a reference interval and finding a rule base[14]

The treatment of the rule base can be realized independently from the initialization of the fuzzy sets. The simulation of the reference interval should run with an autoregressive expectation hypothesis. This implies that people try to find their way in an unknown environment by orientating themselves by the status quo which seems to be a sound (rational) behavior. The fact that any parameter of the fuzzy system can be influenced manually corresponds to the CAL-dimension[15] of MAKROMAT-nfx according to which the user should have the opportunity to become acquainted with the technology by means of individual experimental designs of his/her own.

3.3 Simulation Studies

MAKROMAT-nfx allows to formulate experience- and rule-based expectations by means of the neuro-fuzzy expectations editor (NFE-editor). This software module can be used at any time for observing and modifying the current fuzzy system and the neural learning environment. Fig. 11 gives a first impression of this tool. The main contents of the NFE-editor are organized on four tabs whose order reflects the typical process of fuzzy and neural design: *fuzzifying* crisp values by defining fuzzy sets, generating or modifying the *rule base,* choosing a method for *defuzzifying* the fuzzy inference result set, and finally setting the *neuro* learning options. The rule base for the first simulation study is contained in the file „demo-nfx1 (gP,

[14] For the underlying feature-map-algorithm see KOHONEN (1988), ch. 5, particularly p. 132.

[15] CAL = Computer Assisted Learning.

two-dimensional).nfx“ and can be loaded using the file menu command „Open Expectation Rules“.

Fig. 11 — *Surface of the neuro-fuzzy expectations editor (fuzzifying tab)*

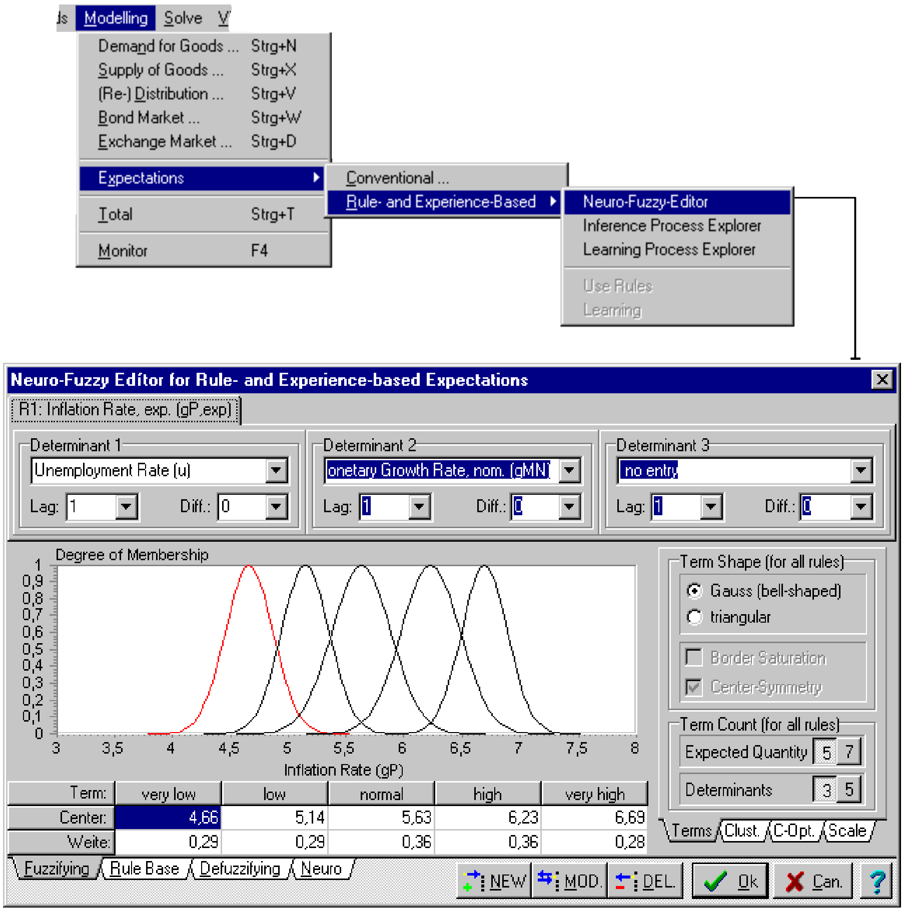
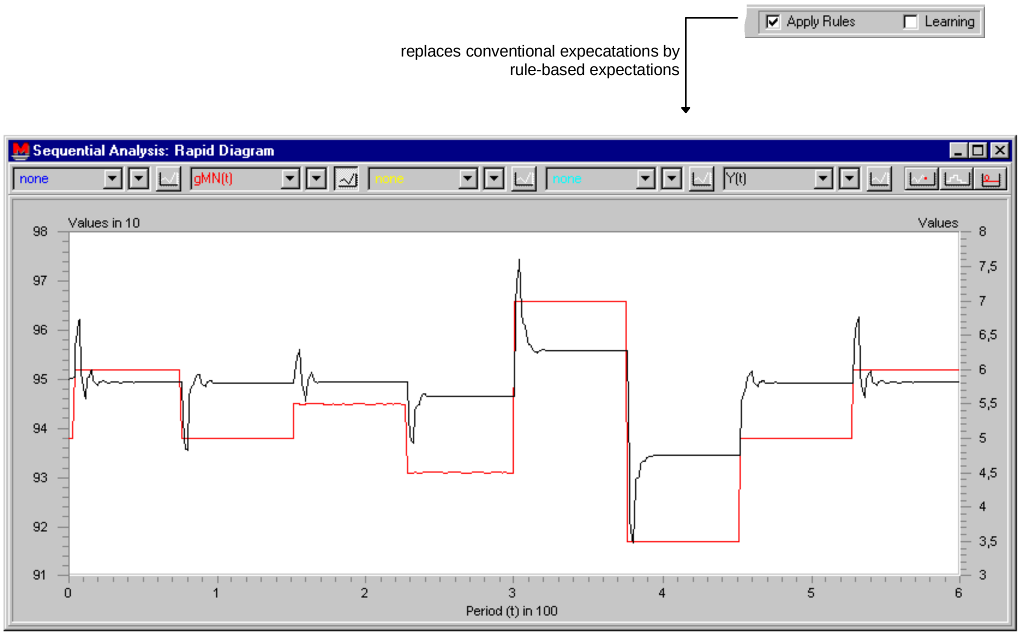


Fig. 12 — *Effect of rule-based inflation expectations without learning*



MAKROMAT-nfx gives us the possibility to simulate rule-based expectation formation with and without the neuro component. For elaborating the influence of the learning option, let us in a first step use fuzzy expectation rules alone. This implies a time-invariant forecasting behavior of the economic units („firm convictions“). In order to realize such an expectation hypothesis the option „Apply Rules“ must be activated. Hereby the former adaptive inflation expectation is replaced by the respective fuzzy rule base and the whole simulation interval is recalculated automatically. Fig. 12 shows the resulting income development for this case.

Fig. 13 — *Effect of rule-based inflation expectations with learning*

Compared with the adaptive expectations version it can be seen clearly that the oscillating behavior of the endogenous time series weakens to a large extent and that the economy stabilizes rather quickly after each monetary shock. However, the realized steady income levels coincide with persistent forecast errors (in the case of correct inflation expectations the equilibrium income level would be equal to the constant natural production level).[16] In order to improve the power of prediction the learning mechanism must be used. Switching it on by activating the „Learning“ option tells the program to apply the (controlled) error-backpropagation algorithm for training the expectation rule system whenever a period has to be recalculated. The recalculation of the whole interval can be released at any time using the „Recalc Interval“ command from the „Solve“ menu. The application of the learning methods leads to the results displayed in Fig. 13.

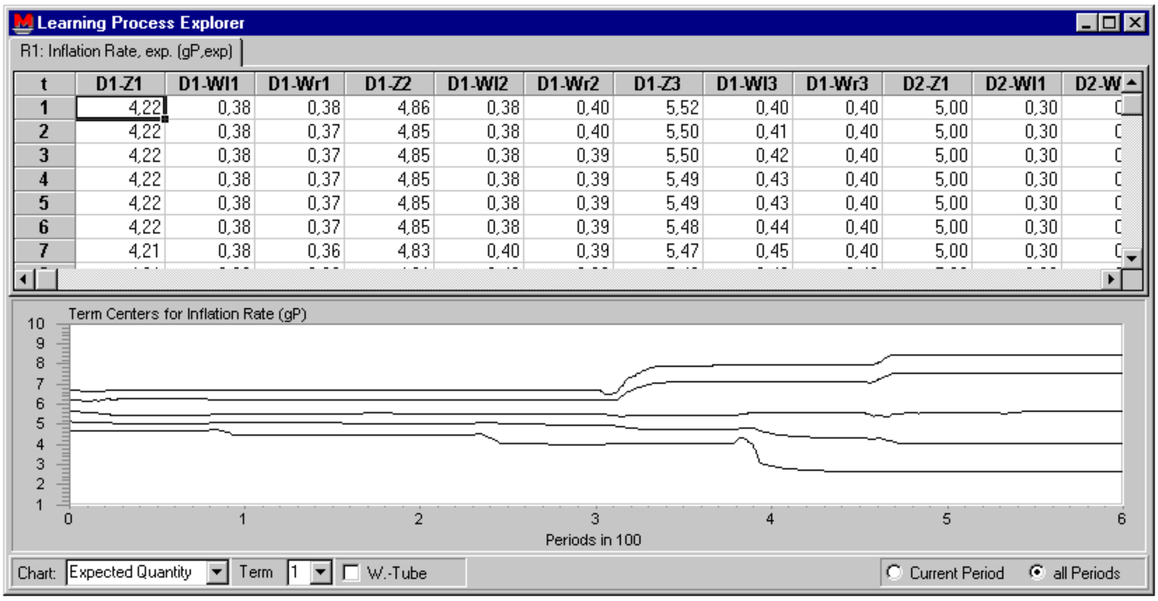
The neural learning procedure reduces forecast errors successively so that the economy stabilizes at the natural level of production. As illustrated in Fig. 13, the adjustment processes responding to strong shifts in monetary policy (periods 300 ff. and 375 ff.) take much more time during the first recalculation round than in the second one („virgin soil effect“). This demon-

[16] To elucidate the problem of false inflation expectations the expected inflation rate (gP^{exp}) could also be displayed in the sequential diagram which was omitted in Fig. 12 for clarity's sake.

strates how the experience with monetary policy influences the forecasting behavior: whereas the monetary maneuvers during the first recalculation round occur completely surprisingly, people can fall back on this experience in the second round and cope much better with them as far as their inflation forecast is concerned.

MAKROMAT-nfx does not only show the effect of experience- and rule-based expectations within the economic simulation model but the program does also include tools which help to understand how they are created. One of these tools is the „Learning Process Explorer" for viewing the development of all fuzzy parameters during the learning process.

Fig. 14 — *Learning process for inflation forecasting (1st calculation round)*



In Fig. 14 the learning process explorer was used to show how people's inflation perception reacts to monetary policy shifts. The so far unusually expansionary monetary policy that starts in period 300 changes the feeling for ‚high' and ‚very high' inflation, so the representative values for these terms (term centers) increase. From 375 on we can observe the same development at the opposite range of inflation estimation as a result of the so far unobserved tight money policy (decrease of the representative values for ‚low' and ‚very low' inflation).

Fig. 15 — *Inference process for inflation forecasting (2^{nd} calculation round, t = 24)*

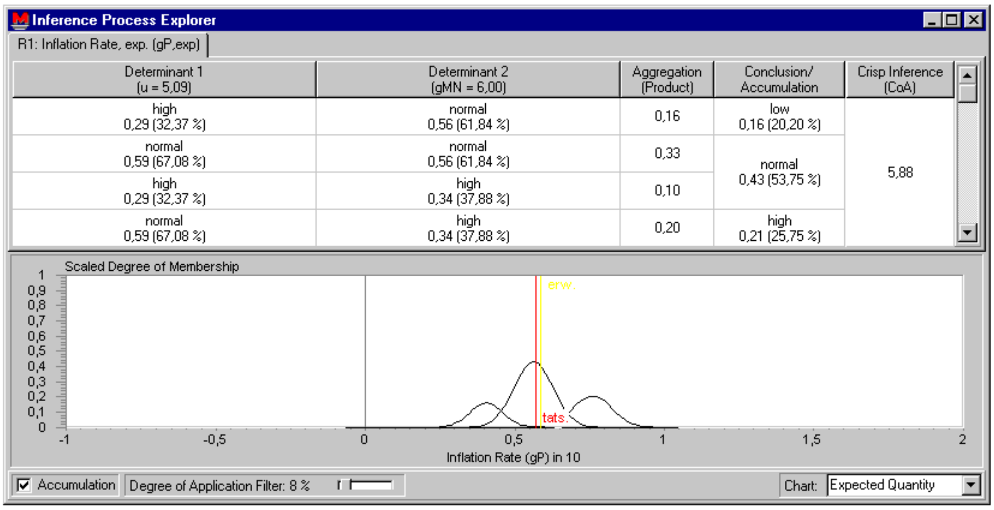
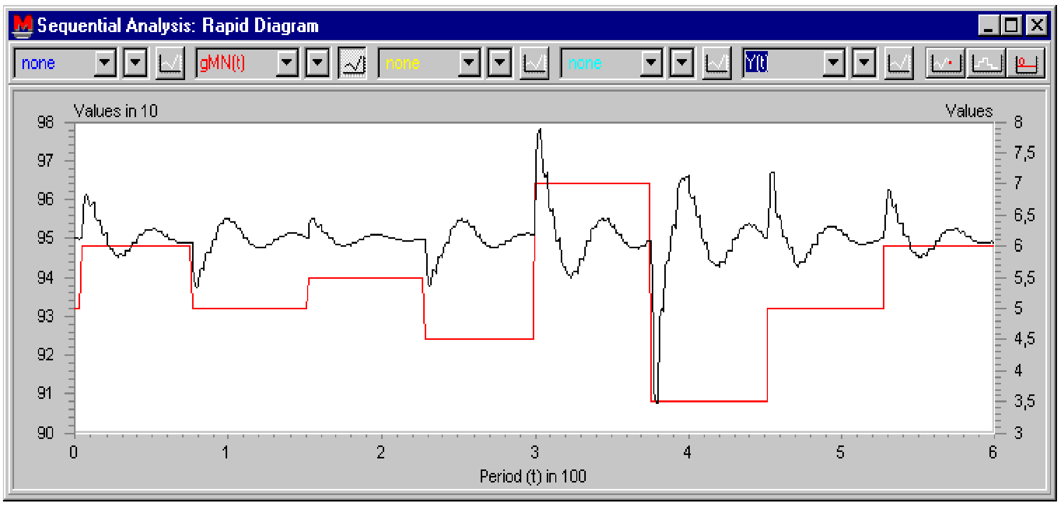


Fig. 15 shows the Inference Process Explorer, which gives an insight into the fuzzy inference process for the current period of the simulation interval. Thanks to the underlying fuzzy system, rule-based expectations can easily be translated into humanly understandable phrases. Let us demonstrate this by verbalizing the first row in the inference table: „Since the observed unemployment rate of 5.09 % is regarded as high to a degree of 0.29 and the money growth rate of 6.0 % is considered normal to a degree of 0.56, future inflation is expected to be low to a degree of 0.16. This explanation influences 20.20 % of the overall inflation forecast.“ In the same way the other table rows can be interpreted which leads to degrees of significance of 0.43 and 0.21 supporting the expectation of normal and high inflation respectively. Put together, we get a compact crisp inflation forecast of gP^{exp} = 5.88 %. This is the representative value of the resulting fuzzy inference result set that is displayed in the inference process explorer's chart zone.

Fig.)



When comparing the 2^{nd} simulation round (Fig. 13, p. 18) with the rational expectations version (Fig. 4, p. 7), one might think that the predictive power of the ERB-expectations is solely due to the second determinant (money growth rate), whereas the unemployment rate seems to be superfluous. This view is refuted by the results shown in Fig. 16. For this simulation a „rational" ERB-version was used by formulating a mono-dimensional rule base with the growth rate of money supply as the single determinant (file: „demo-nfx1 (gP, rational, 3 terms).nfx").

As can be seen from the oscillating income development in Fig. 16 the predictive power of the „rational" rule base is much weaker than in the two-dimensional case. It is also weaker than the conventional rational expectation hypothesis, because the defuzzified inflation forecast value cannot be as exact as the identity between the money supply growth rate and the expected inflation rate. Even the use of a more differentiated partitioning of the relevant interval (use of five instead of three fuzzy sets) does not help significantly. The reader may test this on his/her own (the relevant rule base can be found in the file „demo-nfx1 (gP, rational, 5 terms).nfx").

To sum up this experiment the use of the unemployment rate within the rule base is not irrelevant. Its consideration enables the neuro-fuzzy approach to learn something about the short-term dynamics of the economic system which helps the NFE-generator to cope with monetary shocks rather successfully. This is not possible if a mono-dimensional rule base with the long-term inflation determinant is applied. Furthermore it is much more convincing to suppose that people use a variety of theories and explanations for their forecasting activities than to assume that they rely on a single (long-term) mono-causal theory alone, even if the latter might be proposed as „rational" in economic text books.

4. Applications and Prospects

The simulation has shown that the neuro-fuzzy approach presented in this paper can be used to realize an alternative to conventional expectation hypotheses in macroeconomic models. The experience- and rule-based expectations produced with the NFE-generator imply a forecasting behavior that is characterized by explicit rule orientation (theory foundation), vague formulation (bounded rationality) and learning processes (acquisition of experience).

The weak point of the presented approach at the current stage of development consists in the fact that the theoretical background of the economic units (rule base as relationship between determinants and the expected quantity) has to be given exogenously and that the learning parameters can only be modified by the model designer. So far there is no room for endogenous theory formation (discovery of new relationships) and meta-supervising of the learning behavior by the economic units on their own. Maybe genetic algorithms are a suitable way to extend the level of endogenous contents of the ERB-expectations approach. For this the so far exogenous parameters must be coded in the form of artificial adaptable genes. The single NFE-generator would then be replaced by a population of NFE-generators which differ partly in their characteristics (e.g. considered determinants or learning rate). The chance of survival of each NFE-generator depends on its predictive power. Well forecasting NFEGs can pass their characteristics to the next generation, whereas the poor performing ones vanish.

From an empirical point of view the approach could be used to construct neuro-fuzzy based business cycle indicators. The simulation software MAKROMAT-nfx is prepared for this, because the model-generated time series which are pumped into the NFE-generator can easily be replaced by actual time series. For it the latter must only be available in the form of standard spreadsheets (Excel-compatible). The advantage of such indicators can be seen in their ability of self-documentation because the inferred result is always based on a transparent estimation of the current economic situation and on the application of explicit rules. As far as the learning mechanism is concerned, a test set should be used to prevent the system from overfitting. The quantity that is needed for evaluating the power of forecast could also be used as the survival function in the genetic neuro-fuzzy approaches.

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